

# Denoising and related inverse problems in astrophysics

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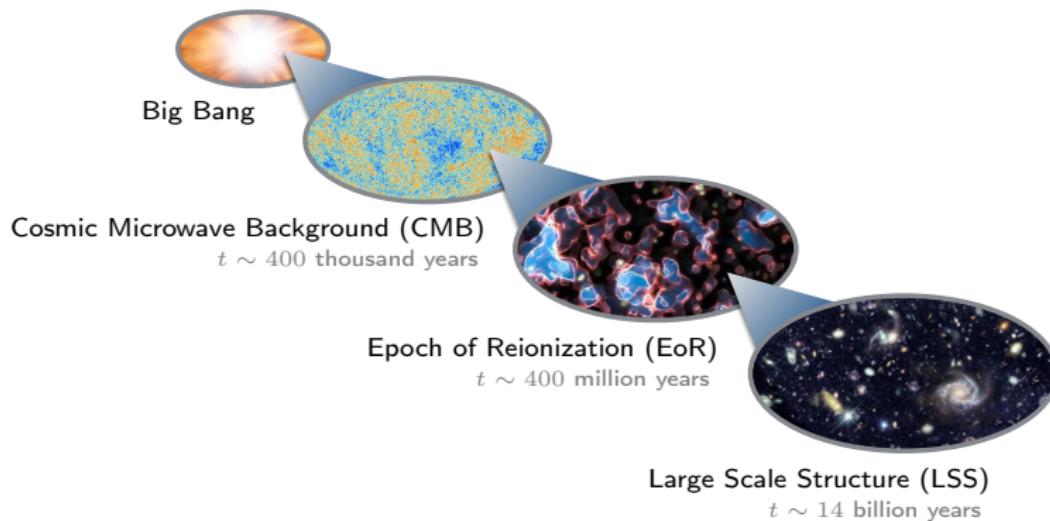
Benchmarking for AI for Science at Exascale (BASE) Workshop  
September 2020

## Signals and noise

"One person's noise is another's signal."

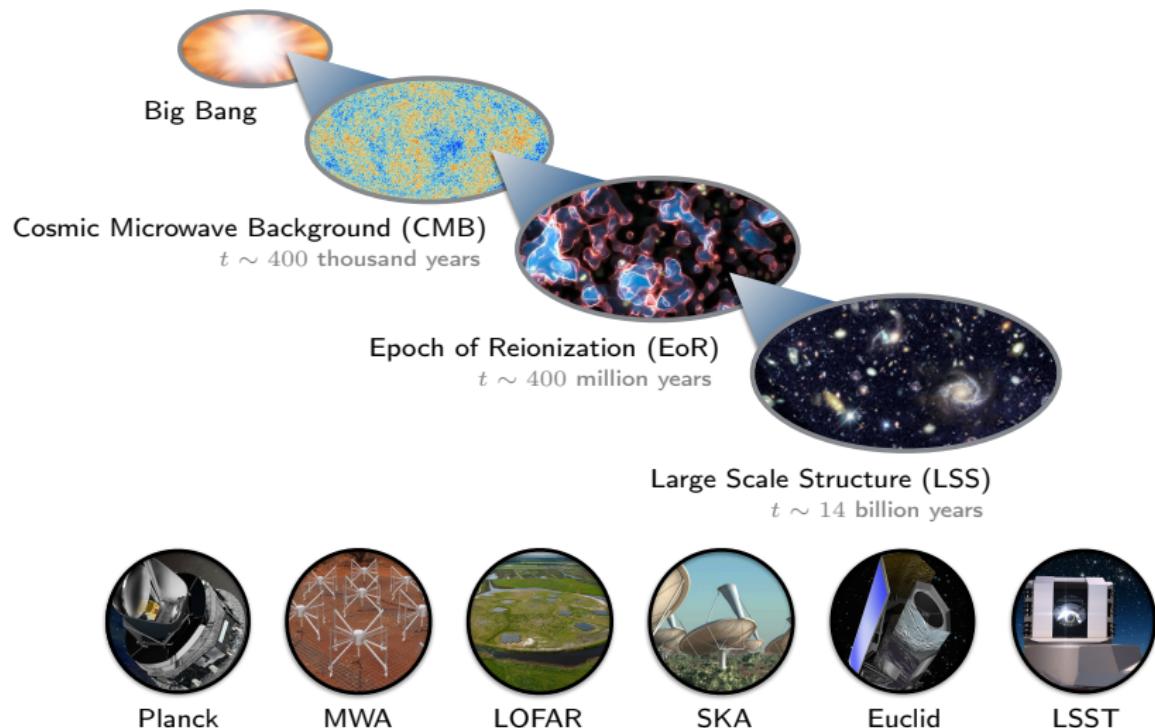
# Cosmic evolution

## Signals and noise



# Cosmic evolution

## Signals and noise



## Denoising and related inverse problems

Ill-posed denoising inverse problem:

$$\mathbf{y} = \mathbf{x} + \mathbf{n} ,$$

where  $\mathbf{y}$  are observations,  $\mathbf{x}$  is the underlying signal of interest, and  $\mathbf{n}$  is noise.

## Denoising and related inverse problems

III-posed denoising inverse problem:

$$\mathbf{y} = \mathbf{x} + \mathbf{n} ,$$

where  $\mathbf{y}$  are observations,  $\mathbf{x}$  is the underlying signal of interest, and  $\mathbf{n}$  is noise.

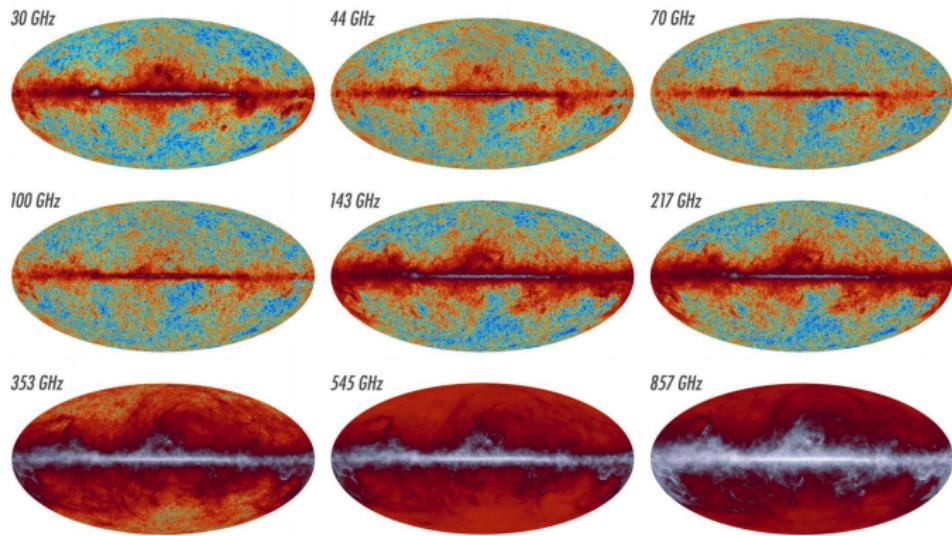
III-posed inverse problem:

$$\mathbf{y} = \Phi \mathbf{x} + \mathbf{n} ,$$

where  $\Phi$  is a linear (measurement) operator.

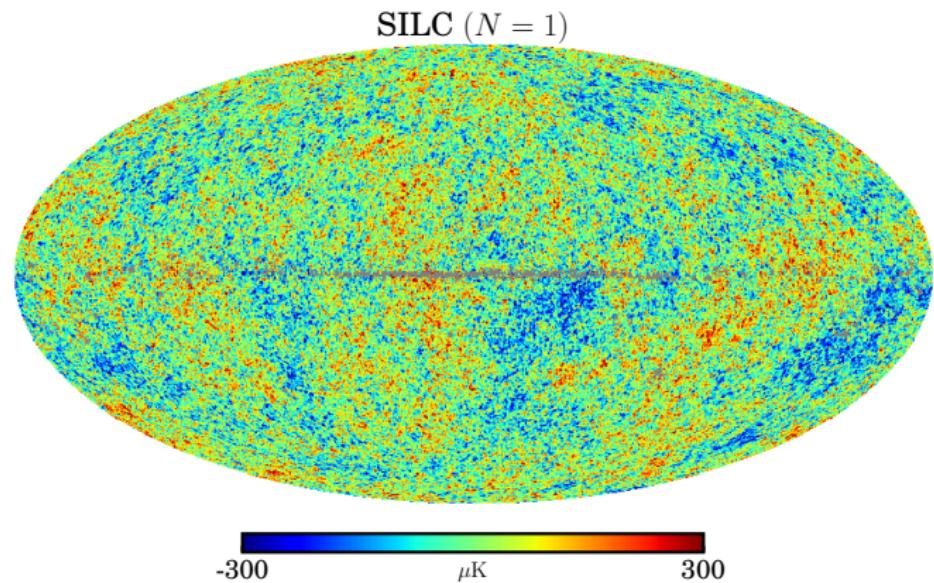
# CMB foreground separation

## Observations at different frequencies



# CMB foreground separation

## Recovered CMB map



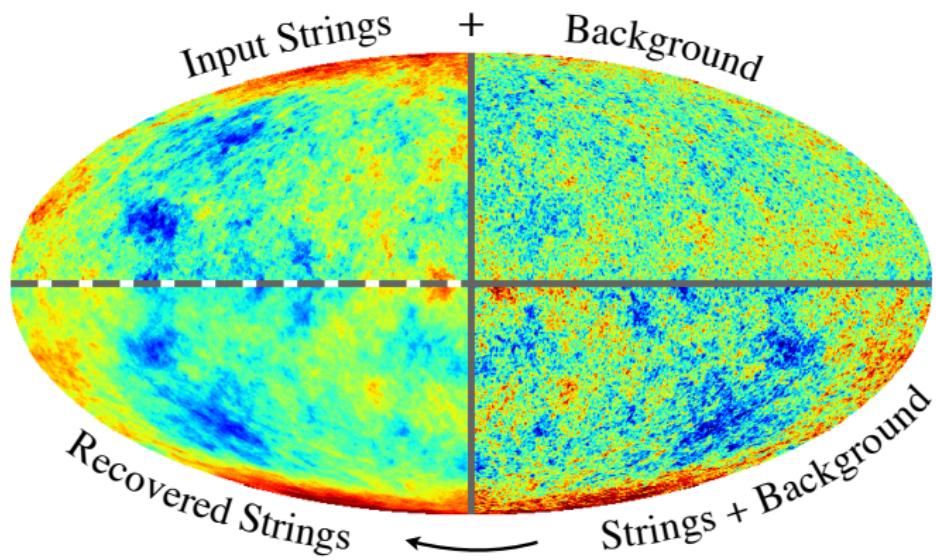
# CMB foreground separation

## Related papers

- Planck Collaboration IV (2018). Planck 2018 results. IV. Diffuse component separation. *Astron. & Astrophys.* [arxiv:1807.06208](https://arxiv.org/abs/1807.06208)
- K. K. Rogers, H. V. Peiris, B. Leistedt, J. D. McEwen, A. Pontzen (2016). Spin-SILC: CMB polarisation component separation with spin wavelets. *Mon. Not. Roy. Astron. Soc.*. [arXiv:1605.01417](https://arxiv.org/abs/1605.01417)
- K. K. Rogers, H. V. Peiris, B. Leistedt, J. D. McEwen, A. Pontzen (2016). SILC: a new Planck Internal Linear Combination CMB temperature map using directional wavelets. *Mon. Not. Roy. Astron. Soc.*. [arXiv:1601.01322](https://arxiv.org/abs/1601.01322)

# Cosmic strings

## Problem



# Cosmic strings

## Hierarchical Bayesian model

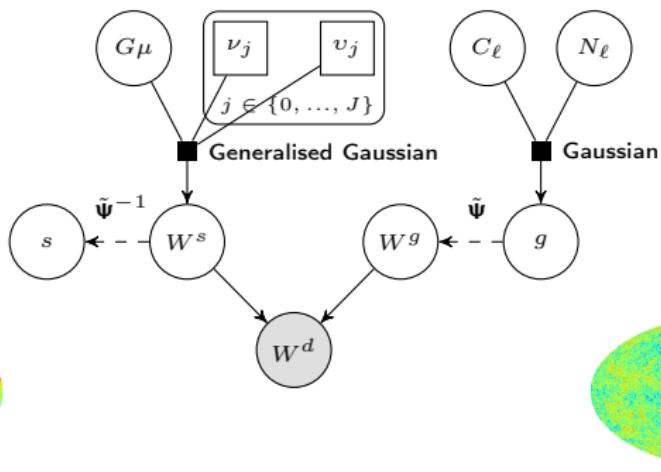


Figure: Hierarchical Bayesian model

# Cosmic strings

## Bayesian inference

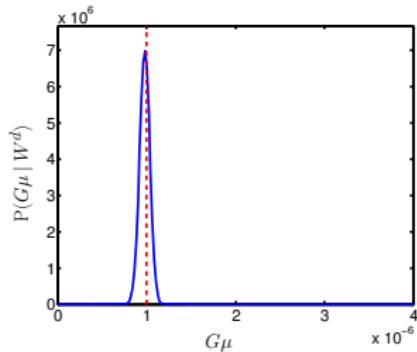
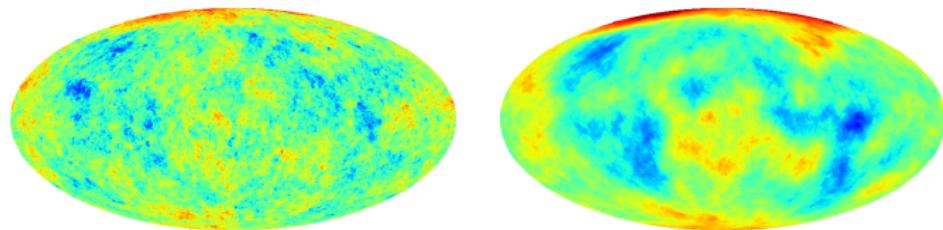


Figure: Posterior

Table: Bayes factors

$G_\mu$ truth / $10^{-7}$	Bayes factor [log <sub>e</sub> ]
10.0	51.4
7.00	12.5
5.00	1.19
3.00	-3.87



(a) Ground truth

(b) Recovered

Figure: String map

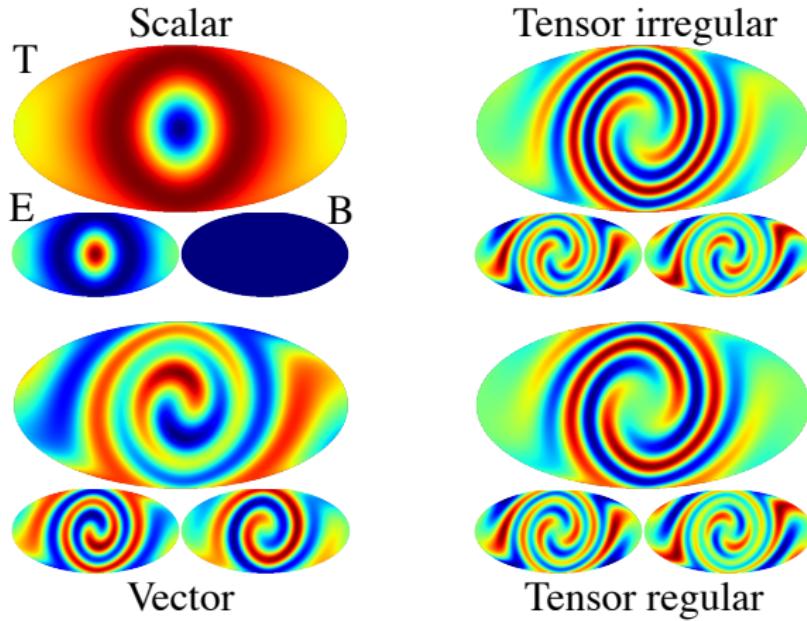
# Cosmic strings

## Related papers

- J. D. McEwen, S. M. Feeney, H. V. Peiris, Y. Wiaux, C. Ringeval, F. R. Bouchet (2017). Wavelet-Bayesian inference of cosmic strings embedded in the cosmic microwave background. *Mon. Not. Roy. Astron. Soc.*. [arXiv:1611.10347](https://arxiv.org/abs/1611.10347)
- Planck Collaboration XXV (2014). Planck 2013 results. XXV. Searches for cosmic strings and other topological defects. *Astron. & Astrophys.* [arXiv:1303.5085](https://arxiv.org/abs/1303.5085)

# Anisotropic cosmologies

## Bianchi models of universal rotation



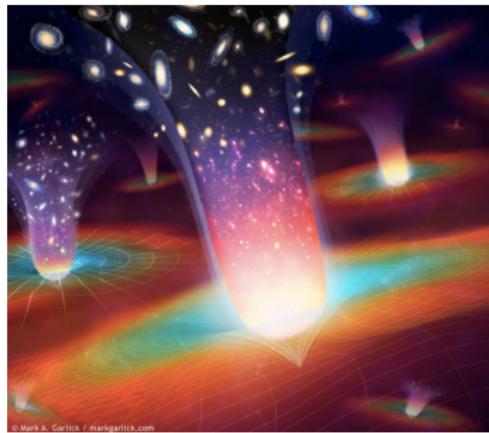
# Anisotropic cosmologies

## Related papers

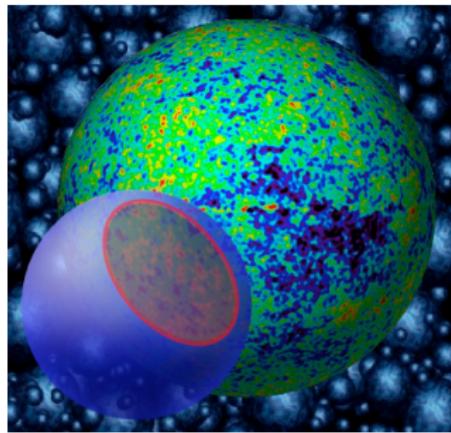
- D. Saadeh, S. M. Feeney, A. Pontzen, H. V. Peiris, J. D. McEwen (2016). How isotropic is the universe?. Phys. Rev. Lett.. [arXiv:1605.07178](#)
- D. Saadeh, S. M. Feeney, A. Pontzen, H. V. Peiris, J. D. McEwen (2016). A framework for testing isotropy with the cosmic microwave background. Mon. Not. Roy. Astron. Soc.. [arXiv:1604.01024](#)
- Planck Collaboration XVIII (2016). Planck 2015 results. XVIII. Background geometry and topology of the Universe. Astron. & Astrophys.. [arXiv:1502.01593](#)
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- J. D. McEwen, T. Josset, S. M. Feeney, H. V. Peiris, A. N. Lasenby (2013). Bayesian analysis of anisotropic cosmologies: Bianchi VII and WMAP. Mon. Not. Roy. Astron. Soc.. [arXiv:1303.3409](#)
- M. Bridges, J. D. McEwen, M. Cruz, M. P. Hobson, A. N. Lasenby, P. Vielva, E. Martinez-Gonzalez (2008). Bianchi VII signatures and the cold spot texture. Mon. Not. Roy. Astron. Soc.. [arXiv:0712.1789](#)
- M. Bridges, J. D. McEwen, A. N. Lasenby, M. P. Hobson (2007). Markov chain Monte Carlo analysis of Bianchi VII models. Mon. Not. Roy. Astron. Soc.. [astro-ph/0605325](#)
- J. D. McEwen, M. P. Hobson, A. N. Lasenby, D. J. Mortlock (2006). Non-Gaussianity detections in the Bianchi VII corrected WMAP 1-year data made with directional spherical wavelets. Mon. Not. Roy. Astron. Soc.. [astro-ph/0510349](#)

# Cosmic bubble collisions and the multiverse

Bianchi models of universal rotation

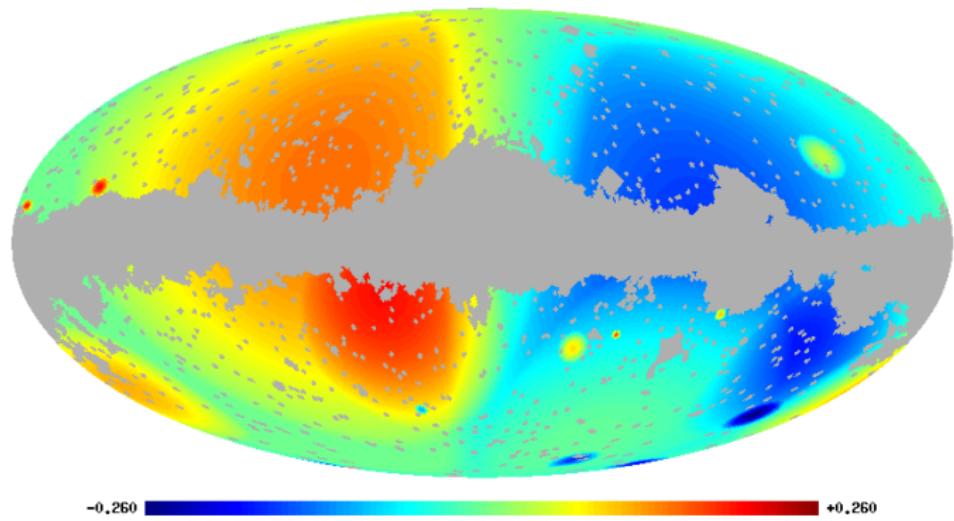


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# Cosmic bubble collisions and the multiverse

Optimal filtering and Bayesian inference



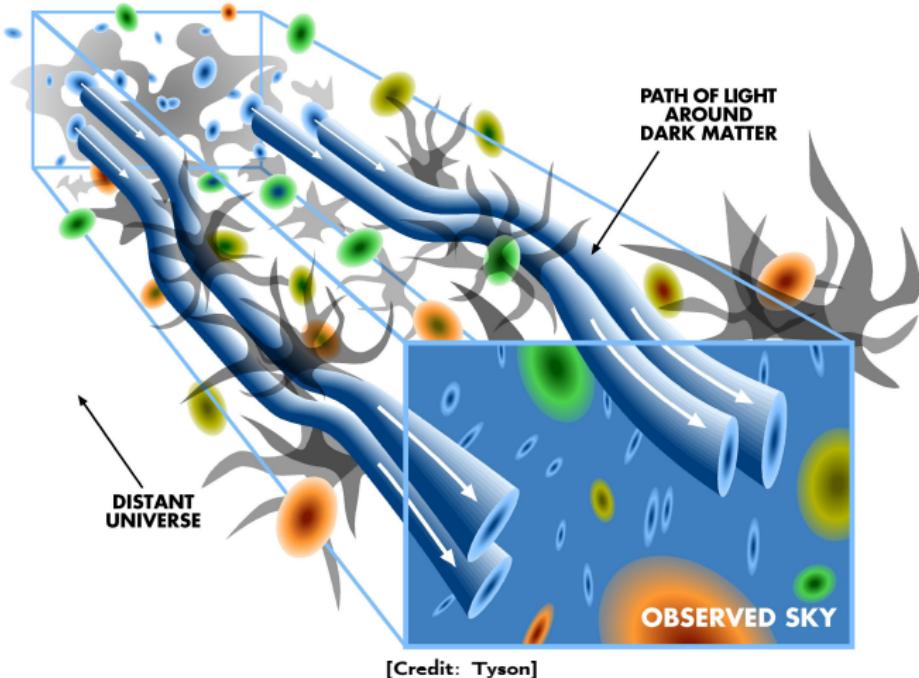
# Cosmic bubble collisions and the multiverse

## Related papers

- S. M. Feeney, M. C. Johnson, J. D. McEwen, D. J. Mortlock, H. V. Peiris (2013). Hierarchical Bayesian detection algorithm for early-Universe relics in the cosmic microwave background. Phys. Rev. D.. [arXiv:1210.2725](https://arxiv.org/abs/1210.2725)
- J. D. McEwen, S. M. Feeney, M. C. Johnson, H. V. Peiris (2012). Optimal filters for detecting cosmic bubble collisions. Phys. Rev. D.. [arXiv:1202.2861](https://arxiv.org/abs/1202.2861)
- J. D. McEwen, M. P. Hobson, A. N. Lasenby (2008). Optimal filters on the sphere. IEEE Trans. Sig. Proc.. [astro-ph/0612688](https://arxiv.org/abs/astro-ph/0612688)

# Mass mapping

## Weak gravitational lensing



# Mass mapping

Mass mapping is a linear inverse problem

- Cosmic shear  ${}_2\gamma$  related to convergence  ${}_0\kappa$  (integrated mass) by:

$${}_2\gamma = 2\partial^2 (\bar{\partial}\bar{\partial} + \bar{\partial}\bar{\partial})^{-1} {}_0\kappa$$

Differential form

$${}_2\gamma(\mathbf{n}) = \int_{\mathbb{S}^2} d\Omega(\mathbf{n}') {}_0\kappa(\mathbf{n}') (\mathcal{R}_{\mathbf{n}} {}_2\mathcal{K})(\mathbf{n}')$$

Integral form

- Mass mapping is a **spherical inverse problem**.
- Solve mass mapping problem in spherical setting, avoiding planar approximations (e.g. Wallis *et al.*).



# Mass mapping

## Related papers

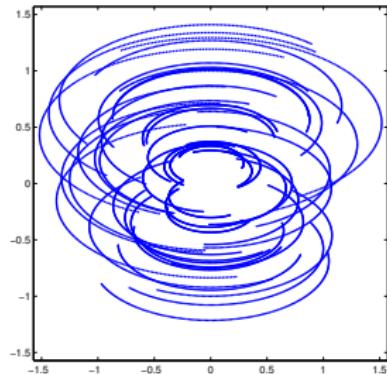
- M. A. Price, J. D. McEwen, L. Pratley, T. D. Kitching (2020). Sparse Bayesian mass-mapping with uncertainties: full-sky observations on the celestial sphere. *Mon. Not. Roy. Astron. Soc.*, submitted. [arXiv:2004.07855](https://arxiv.org/abs/2004.07855)
- M. A. Price, J. D. McEwen, X. Cai, T. D. Kitching (2019). Sparse Bayesian mass-mapping with uncertainties: peak statistics and feature locations. *Mon. Not. Roy. Astron. Soc.*. [arXiv:1812.04018](https://arxiv.org/abs/1812.04018)
- M. A. Price, X. Cai, J. D. McEwen, M. Pereyra, T. D. Kitching (2019). Sparse Bayesian mass-mapping with uncertainties: local credible intervals. *Mon. Not. Roy. Astron. Soc.*. [arXiv:1812.04017](https://arxiv.org/abs/1812.04017)
- M. A. Price, J. D. McEwen, X. Cai, T. D. Kitching, C. G. R. Wallis (2019). Sparse Bayesian mass-mapping with uncertainties: hypothesis testing of structure. *Mon. Not. Roy. Astron. Soc.*, submitted. [arXiv:1812.04014](https://arxiv.org/abs/1812.04014)
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# Radio interferometric imaging

## Observational process



“Fourier”  
Measurements



# Radio interferometric imaging

## Compressive sensing and sparse regularisation

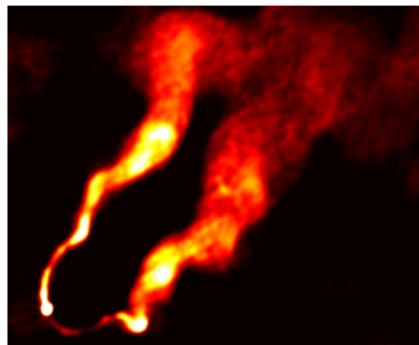
- Compressed sensing motivates **sparse regularisation**, imposing sparse prior in some representation  $\alpha$  (e.g. wavelets), where  $x = \Psi\alpha$ :

$$\alpha^* = \arg \min_{\alpha} \|\alpha\|_1 \text{ s.t. } \|y - \Phi\Psi\alpha\|_2 \leq \epsilon$$

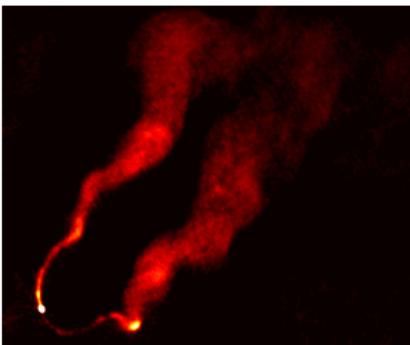
Synthesis framework

# Radio interferometric imaging

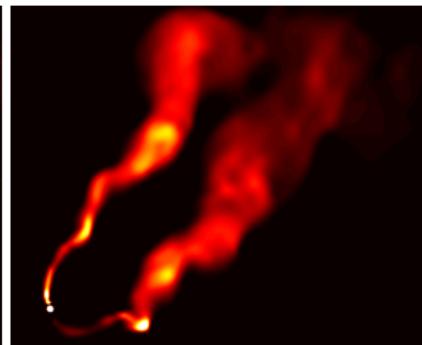
PURIFY reconstruction of observations of 3C129 by the VLA



(a) CLEAN (natural); DR=220



(b) CLEAN (uniform); DR=495

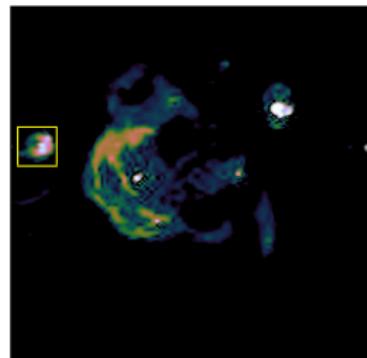


(c) PURIFY; DR=969

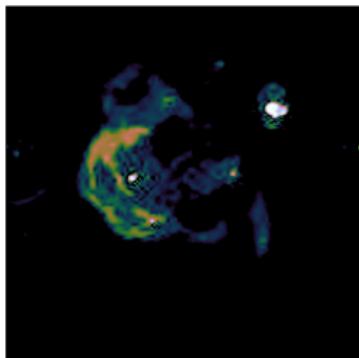
Figure: 3C129 (Pratley, McEwen, et al. 2016)

# Radio interferometric imaging

## Uncertainty quantification



(a) Recovered image



(b) Surrogate with region removed

Reject null hypothesis  
⇒ structure physical

Figure: Supernova remnant W28

# Radio interferometric imaging

## Deep learning

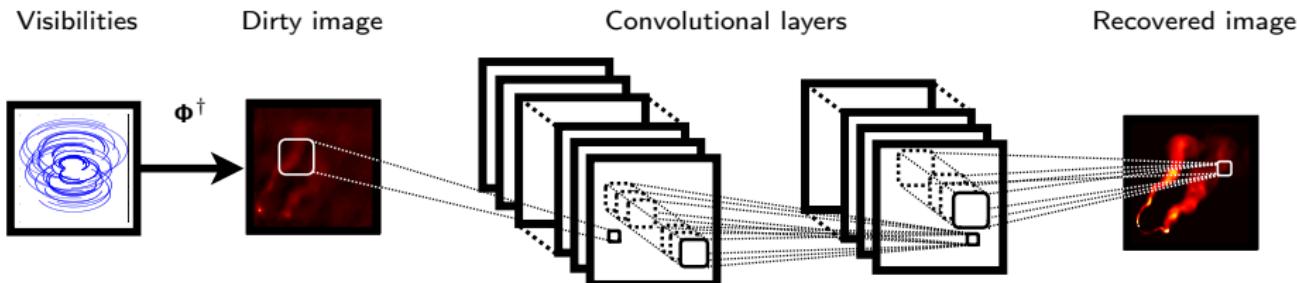


Figure: Deep learning architecture for interferometric imaging (Allam & McEwen, 2016)

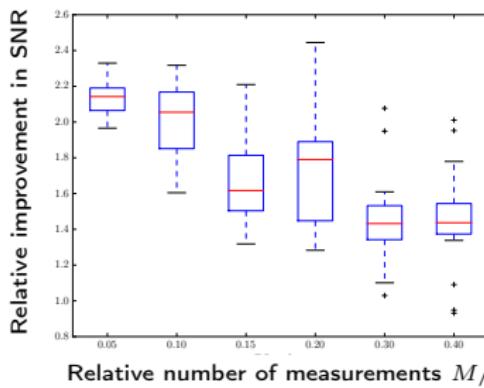
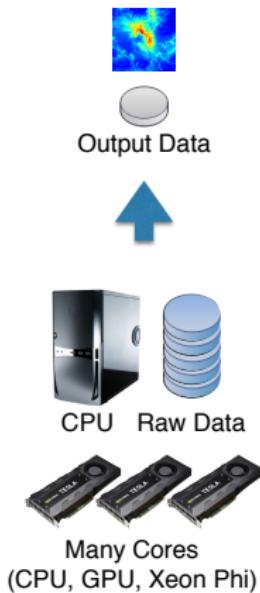


Figure: Improvement in signal-to-noise-ratio (SNR)

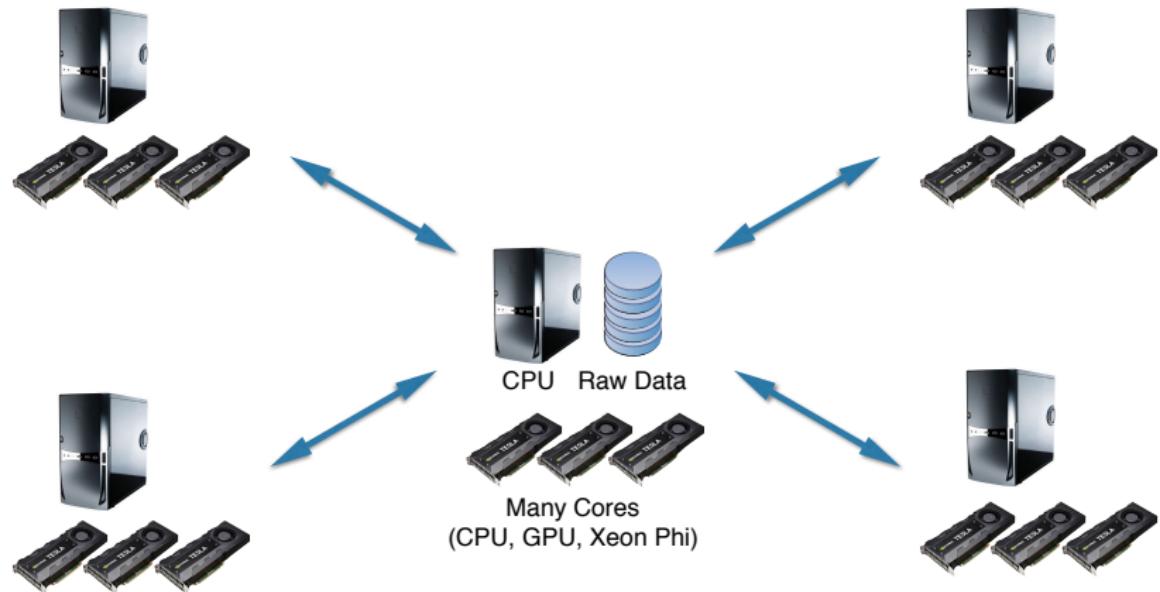
# Radio interferometric imaging

## Standard algorithms



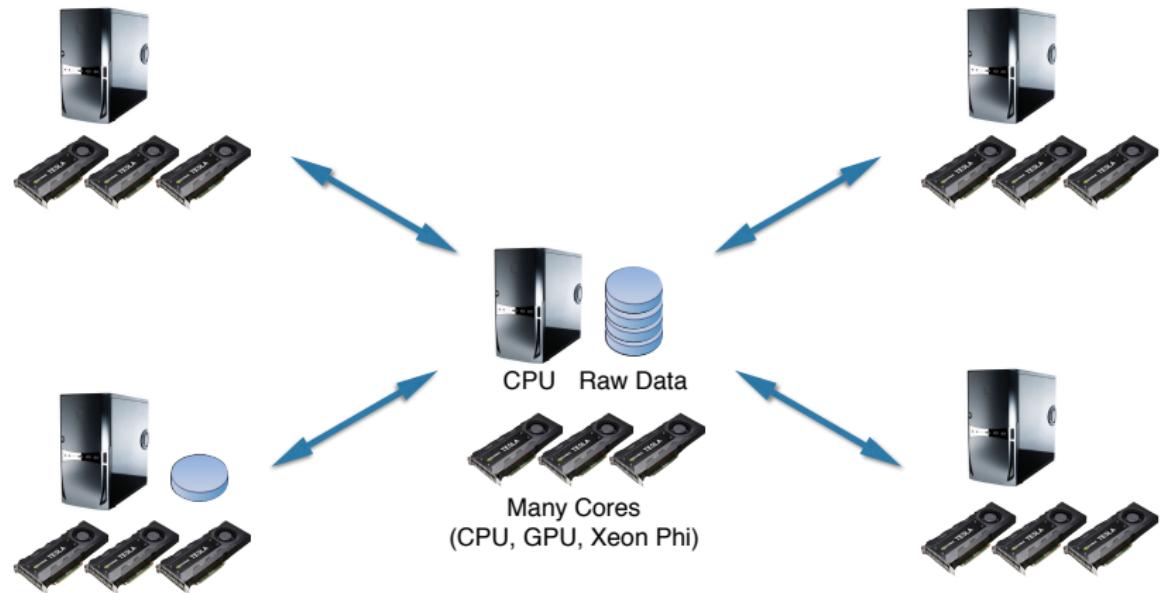
# Radio interferometric imaging

## Highly distributed and parallelized algorithms



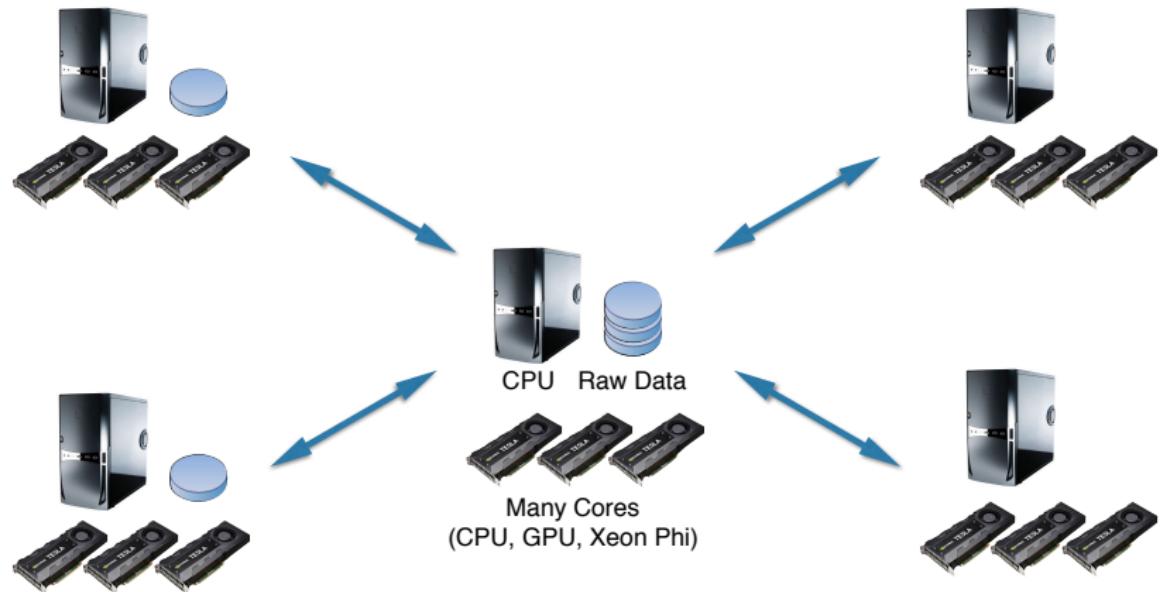
# Radio interferometric imaging

## Highly distributed and parallelized algorithms



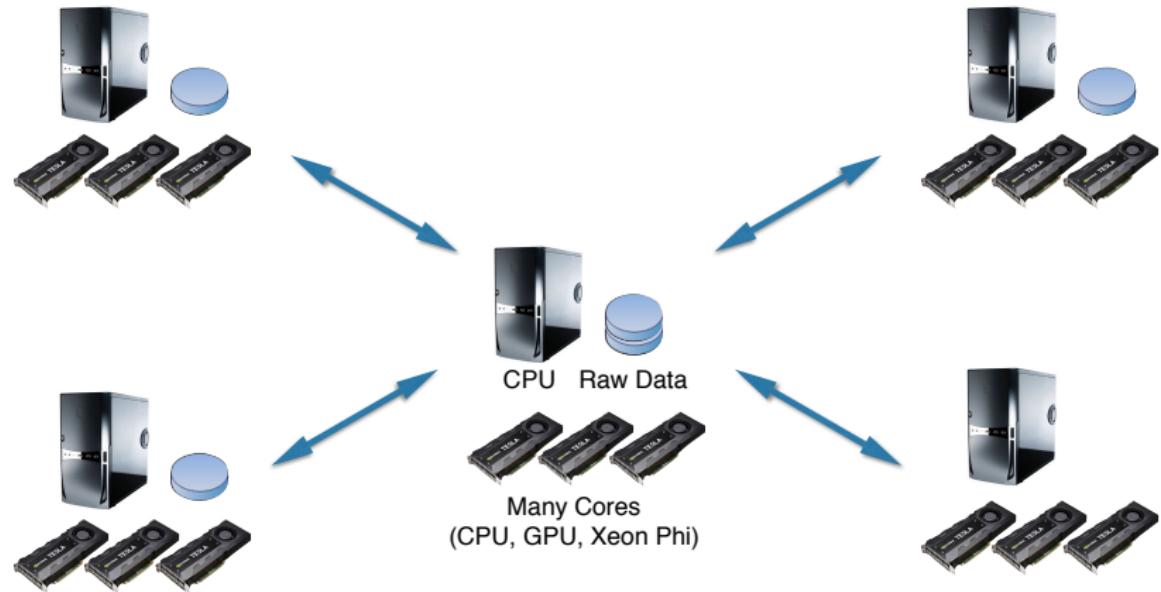
# Radio interferometric imaging

## Highly distributed and parallelized algorithms



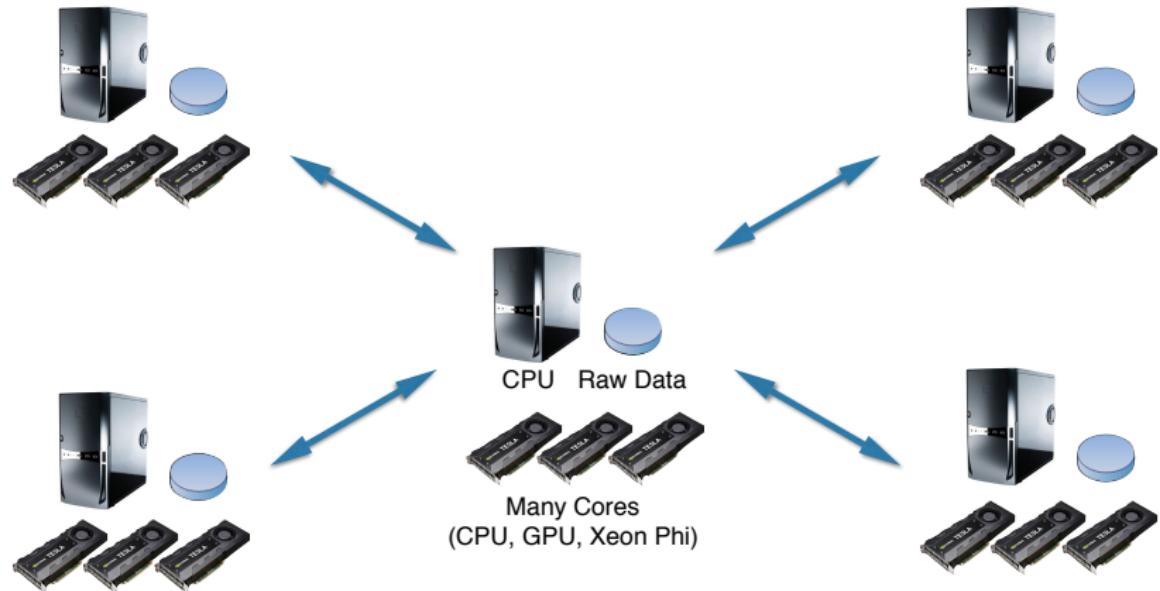
# Radio interferometric imaging

## Highly distributed and parallelized algorithms



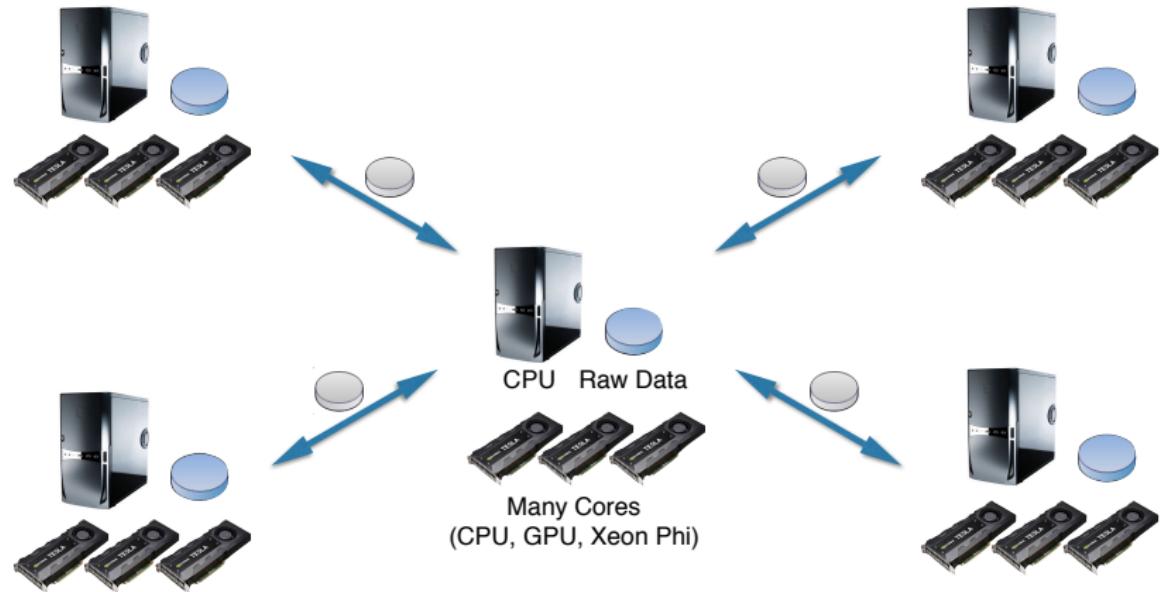
# Radio interferometric imaging

## Highly distributed and parallelized algorithms



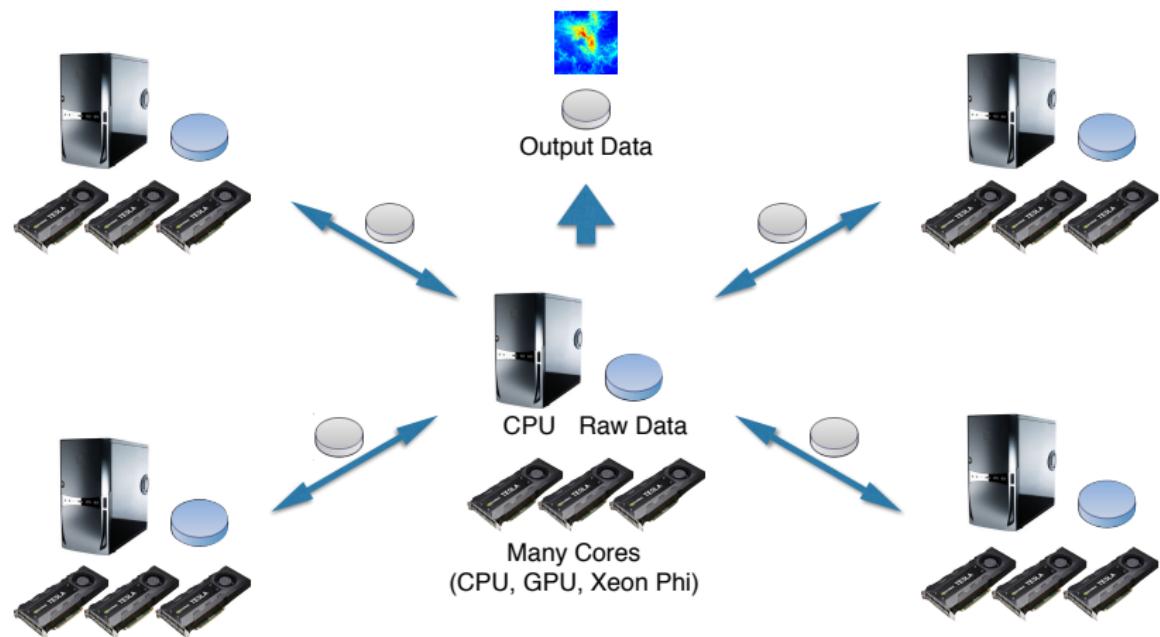
# Radio interferometric imaging

## Highly distributed and parallelized algorithms



# Radio interferometric imaging

## Highly distributed and parallelized algorithms



# Radio interferometric imaging

## Related papers

- X. Cai, L. Pratley, J. D. McEwen (2020). Offline and online reconstruction for radio interferometric imaging. XXXIVth General Assembly and Scientific Symposium of the International Union of Radio Science. <https://arxiv.org/abs/arXiv:2004.06478>
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- L. Pratley, J. D. McEwen (2019). Load balancing for distributed interferometric image reconstruction. *Mon. Not. Roy. Astron. Soc.*, submitted. <https://arxiv.org/abs/arXiv:1903.07621>
- L. Pratley, J. D. McEwen, M. d'Avezac, X. Cai, D. Perez-Suarez, I. Christidi, R. Guichard (2019). Distributed and parallel sparse convex optimization for radio interferometry with PURIFY. *Astron. Comput.*, submitted. <https://arxiv.org/abs/arXiv:1903.04502>
- L. Pratley, M. Johnston-Hollitt, J. D. McEwen (2019). w-stacking w-projection hybrid algorithm for wide-field interferometric imaging: implementation details and improvements. *Mon. Not. Roy. Astron. Soc.*, submitted. <https://arxiv.org/abs/arXiv:1903.06555>
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<https://arxiv.org/abs/arXiv:1712.04462>
- C. Skipper, A. M. M. Scaife, J. D. McEwen (2019). Cleaning radio interferometric images using a spherical wavelet decomposition. *Astron. Comput.*. <https://arxiv.org/abs/arXiv:1909.03956>

# Radio interferometric imaging

## Related papers

- L. Pratley, M. Johnston-Hollitt, J. D. McEwen (2019). A fast and exact w-stacking and w-projection hybrid algorithm for wide-field interferometric imaging. *Astrophys. J.*. [arXiv:1807.09239](https://arxiv.org/abs/1807.09239)
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# Radio interferometric imaging

## Related papers

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# Radio interferometric imaging

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- L. Wolz, F. B. Abdallah, R. E. Carrillo, Y. Wiaux, J. D. McEwen (2013). The varying- spread spectrum effect for radio interferometric imaging. Biomedical and Astronomical Signal Processing Frontiers (BASP). [arXiv:1301.7259](https://arxiv.org/abs/1301.7259)
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- L. Wolz, J. D. McEwen, F. B. Abdalla, R. E. Carrillo, Y. Wiaux (2013). Revisiting the spread spectrum effect in radio interferometric imaging: a sparse variant of the -projection algorithm. Mon. Not. Roy. Astron. Soc.. [arXiv:1307.3424](https://arxiv.org/abs/1307.3424)
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- J. D. McEwen, Y. Wiaux (2011). Compressed sensing for wide-field radio interferometric imaging. Mon. Not. Roy. Astron. Soc.. [arXiv:1010.3658](https://arxiv.org/abs/1010.3658)

# Radio interferometric imaging

## Related papers

- J. D. McEwen, Y. Wiaux (2011). Compressed sensing for radio interferometric imaging: review and future direction. 18th IEEE International Conference on Image Processing (ICIP), invited contribution. [arXiv:1110.6137](https://arxiv.org/abs/1110.6137)
- J. D. McEwen, A. M. M. Scaife (2008). Simulating full-sky interferometric observations. Mon. Not. Roy. Astron. Soc.. [arXiv:0803.2165](https://arxiv.org/abs/0803.2165)