Wide-field interferometric imaging via distributed sparse image reconstruction

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***Abstract*—Low frequency radio interferometric telescopes, made from non-coplanar antenna arrays, have extremely wide-fields of view ($\sim 30^\circ$). To perform wide-field imaging, the w-projection algorithm models wide-fields of view with the non-coplanar w-term. However, calculating the exact model for each measurement has not been possible due to the amount of computation required at high resolution and with the large volume of data from current interferometers. The required accuracy and computational cost of these corrections is one of the largest unsolved challenges facing next generation radio interferometers such as the Square Kilometre Array. We show that the same calculation can be performed with a radially symmetric w-projection kernel, where we use one dimensional adaptive quadrature to calculate the resulting Hankel transform, decreasing the computation required for kernel generation by several orders of magnitude, whilst preserving the accuracy. We demonstrate the potential of our radially symmetric w-projection kernel via sparse image reconstruction, using the software package PURIFY. We develop an MPI distributed w-stacking and w-projection hybrid algorithm, where we apply exact w-term corrections for 100 million measurements.**

I. Introduction

Next-generation low frequency radio interferometers, such as the Square Kilometer Array (SKA-LOW1; [1]), must meet the challenge of processing and imaging extremely large volumes of data to realize high-profile science goals, including detecting the first stars [2]. Radio interferometric antenna arrays allow higher resolution and sensitivity than possible with a single antenna telescope, at the computational cost reconstruction due to incomplete sampling of measurements in the Fourier domain (u, v, w). If the science goals of SKA are to be achieved, state of the art methods in image reconstruction are critical to process the big data and to reconstruct images with high fidelity.

In [3], the proximal alternating direction method of multipliers algorithm (ADMM) developed by [4] was implemented in the software package PURIFY, where ℓ_1-regularization was applied to observational data. PURIFY uses a non-uniform fast Fourier transform (NUFFT, also known as gridding/degridding) as a measurement (equation) operator and a Daubechies wavelet dictionary as a sparsity operator.

However, for a telescope with wide-fields of view, the measurement operator needs to model the impact of the out of plane component of the array known as the w-term. The w-term can be modeled as multiplying the image with a measurement dependent linear chirp in the image domain $x(l,m)\mathrm{e}^{-2\pi i w(\sqrt{1-l^2-m^2}-1)}$, where $x(l,m)$ is the sky brightness. This measurement dependent multiplication can be incorporated as a convolution with the interpolation kernels (anti-aliasing filter) during the NUFFT, and is known as the w-projection algorithm.

Typically, these w-projection kernels are modeled by performing an FFT for each w-term. This makes exact correction of the w-term impossible for extremely wide-fields of view at high resolution. We propose a solution to this challenge in [5]. Since the linear chirp is smooth and has an analytic form, we propose to use adaptive quadrature to evaluate the two dimensional (2d) convolution kernel

$$[GC](u_{\rm pix}, v_{\rm pix}, w, \Delta u, \Delta v) = \int_{-\alpha/2,-\alpha/2}^{\alpha/2,\alpha/2} \Theta(1 - x^2/\Delta u^2 - y^2/\Delta v^2) \times \mathrm{e}^{-2\pi i w(\sqrt{1-x^2/\Delta u^2 - y^2/\Delta v^2}-1)} g(x)g(y) \times \mathrm{e}^{-2\pi i(u_{\rm pix}x + v_{\rm pix}y)}\mathrm{d}x\mathrm{d}y\,. \tag{1}$$

α is the oversample ratio of the NUFFT, $(\Delta u, \Delta v)$ is the 2d resolution of the FFT grid, (x, y) are the coordinates of the image.

We show that when $\Delta u = \Delta v$, there is radially symmetry of the linear chirp. It then follows that using a radial anti-aliasing kernel reduces the quadrature calculation to a one dimensional (1d) integral

$$[GC](\sqrt{u_{\rm pix}^2 + v_{\rm pix}^2}, w, \Delta u) = 2\pi \int_0^{\min\{\alpha/2, \Delta u\}} g(r) \times \mathrm{e}^{-2\pi i w(\sqrt{1-r^2/\Delta u^2}-1)} J_0\left(2\pi r\sqrt{u_{\rm pix}^2 + v_{\rm pix}^2}\right) r\mathrm{d}r\,. \tag{2}$$

We demonstrate that this calculation is as accurate as the 2d quadrature calculation, but requires less samples for convergence. If the 1d integral requires N samples, the 2d integral requires N^2 samples.

We implemented the 1d integration in PURIFY. We also use the message parsing interface (MPI) on a computing cluster, allowing for a distributed ADMM described in [6], including distributed proximal operators, wavelet operators, and measurement operators. This distributed structure allows natural implementation of a w-stacking algorithm, where measurements are clustered into average w groups (i.e. using k-means), one group for each node. This allows for an average w correction to be applied as the linear chirp in the image domain for each node. Then the residual chirp can be corrected using the w-projection method. This allowed us to apply exact w-term corrections for 100 million measurements with w-terms between ± 300 (radio) wavelengths, for a 17° field of view and image size of 4096 by 4096 pixels. The runtime was 35 minutes on 50 nodes.

References

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